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How Does It Happen?

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Students Know the Concept but are Incorrect in Solving the Proportional Problem: How Does It Happen?

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Abstract: It is difficult for students to solve proportion problems. This case study describes how students solve proportion problems by taking heed of mental mechanisms, namely interiorization, coordination, encapsulation, and generalization. The subjects in this study were the seventh graders of junior high schools in Yogyakarta, Indonesia. The subjects were chosen purposively, namely paying attention to subjects' mistakes in solving proportion problems. The findings show that students undergo problems in the division, and they are unable to formulate the right formula even though they know the concept of proportions, mixed solutions to direct proportion and inverse proportion

Keywords: Proportional Concept, Mistakes, Problem-solving

Introduction

Proportional reasoning is used to understand and solve proportion problems. Proportion reasoning is a topic that has attracted the attention of many researchers (Ekawati, Lin, and Yang 2015; Sumarto et al. 2014; Van Dooren et al. 2009). The concept of proportion is fundamentally used to understand other mathematical concepts, including percentage, algebra, and trigonometry (Doyle et al. 2015; Jitendra et al. 2016). Thus, proportional reasoning is important for students to understand.

However, research reveals that many students find it difficult to solve proportion problems. Students sometimes use the concept of addition to solve proportion problems (Fernández et al. 2012). Likewise, students use the concept of proportion to solve non-proportion problems (Irfan et al. 2019; Irfan, Sudirman, and Rahardi 2018; Van Dooren et al. 2009). In addition, students often experience interferences when they deal with direct proportion and inverse proportion (De Bock, Van Dooren, and Verschaffel 2013; Irfan et al. 2018).

One of the causes of students' errors in resolving proportional problems is a weak understanding of proportion concepts. Some studies have found that the concept of proportion can be constructed from several materials, including numbers and fractions changing from story problems to mathematical models, division, multiplication (Bayazit 2013; Doyle et al. 2015; Fatimatul Khikmiyah, Agung Lukito 2012). Furthermore, the role of textbooks and teachers' teaching strategies contributes to the conception of students' proportional understanding (Bayazit 2013; Ekawati, Lin, and Yang 2015; Jitendra et al. 2016; Lemonidis 2008).

Proportion problems are an interesting topic for researchers. For example, some research-based textbooks are used in schools (Bayazit 2013; De la Cruz 2013; Ekawati, Lin, and Yang 2015), which include strategies on how teachers and prospective teachers teach (Arican 2016; Livy and Herbert 2013; Lobato et al. 2011) and students' errors in the processes of resolving

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proportion problems (De Bock, Van Dooren, and Verschaffel 2013; Irfan et al. 2019; Subanji 2013; van Dooren et al. 2010; Modestou et al. 2008). Conversely, only a few researchers discuss the construction of proportional concepts. When students experience errors in solving problems, they are possibly unable to use the knowledge they have learned (Van Hoof et al. 2013) because certain parts (of materials) are difficult to understand. It is interesting for researchers to explore how students think and make decisions when solving proportion problems.

This research contributes to literature in terms of (1) adding references to students' errors when solving proportion problems, and (2) providing guidance to improve teaching strategies to minimize students' errors in solving proportion problems.

Proportion

The concept of ratio is a multiplicative relationship between two values calculated by dividing (or multiplying) one quantity with another. Students have learned the concept of ratios since elementary school, although the term proportion or ratio is not explicitly introduced to students. The ratio is a comparison between two quantities (Lamon 2006; Livy and Herbert 2013; Silvestre and da Ponte 2011) that can be represented by fractions, and the fraction rules can be applied to the ratio. The initial concept of the ratio can be given to students so that they are familiar with the fractions and operations. Ratios are a basis for understanding concepts of proportions (Ben-Chaim, Keret, and Ilany 2007; Van Dooren, Lehtinen, and Verschaffel 2015). There are three general proportion, namely ratio, parts (for example, one person compared to 3 balls or 1: 3); part-whole (for example, two out of seven parts or 2/7); and scaling, whole-whole (comparing wholeness with wholeness, where 1cm on the map equals 1,000,000 cm on the ground) (Doyle et al. 2015; Parish 2010; Fatimatul Khikmiyah, Agung Lukito 2012). The proportion is a relationship between four numbers, or numbers in which the ratio of the first pair is equal to the ratio of the second pair written as a: b = c: d (Boyer and Levine 2012; Son 2013). Proportion problems involve situations in which mathematical relationships are multiplicative (as opposed to additives), which allows the formation of the same ratio (Ben-Chaim, Keret, and Ilany 2007).

Direct proportion and inverse proportions are part of proportional relations. The difference between these proportions is indicated by the direction of change, whether or not it is the same or the opposite. This is called a direct proportion whereas a change occurs in the same direction or it is called an inverse direction when changes go in a different direction. The scale of the map and the relationship between distance and fuel requirements are examples of comparable values. An example of an inverse proportion is as follows: there are four workers, the time spent to renovate a house is 24 days, how long will the work be done if the number of workers increases to 6?

Method

This is a qualitative case study. The cases in question were the seventh graders of junior high school students in Yogyakarta who found difficulties in generating proportional concepts.

Participants

The participants in this research were thirty-two seventh graders in the second semester at public junior high schools in Yogyakarta. The research subjects were chosen by considering the results of students' assignments when completing three quizzes, namely quizzes on direct proportion, inverse proportion, and both types of proportion. Researchers gave the quiz at the end of the meeting. The time allotment to answer each quiz is fifteen minutes. Of the total students who answered the quizzes, eight could answer correctly and twenty-four answered incorrectly. The classification of students' answers can be viewed in Table 1.

Table 1: The Classification of Students' Answers

Type Quiz	*Correct*	*Incorrect*
Direct proportion	26	6
Inverse proportion	15	17
Both	18	14

Source: Irfan et al.

From students who answered incorrectly, various types of errors occurred, including miscalculation, incomplete work, interference (exchanged comparable concepts of direct proportion and inverse proportion), and the inability to choose the method of completion.

Data Collection

The data in this research were collected using test instruments (quizzes) and interviews. Test instruments were used to see students' understanding after learning from the teacher, while interviews were used to explore how students think when completing the quiz. Subjects were selected based on the answers of each quiz that meet the criteria: the inability to choose a method of completion or a direct proportion concept to solve the problem of direct proportion, or vice versa. This is because researchers want to discuss how students generate the concept of proportion in every material that has been taught through problem solving in each quiz.

Analysis

The process of data analysis was taken through six steps (Creswell 2012): (1) the transcoding of data collected. In this sense, the data transcribed were the results of tests and interviews with the subjects. (2) Reviewing the available data from test results and interview transcripts. (3) Reducing data by selecting, focusing on, and classifying the similar data, and simplifying them by removing unnecessary things. The researcher selected the data generated from the tests in line with predetermined indicators. (4) Presenting data on research results. In this step, the researchers presented the results of the study from tests of students who found it difficult to solve the proportion problem. (5) Analyzing the process of understanding the concept of proportion based on the results of students' answers. And (6), verifying the findings and drawing conclusions. The researchers verified the findings, in this case, an understanding of the concept of proportion used to draw conclusions. This research is intended to investigate students in constructing the knowledge and understanding of the concept of proportion. To investigate the way students construct their knowledge, the APOS framework was developed by Dubinsky. In the APOS theory, the knowledge formation includes mental structures and mental mechanisms (Arnon et al. 2014). The concept of proportion is generated through mental mechanisms, namely interiorization, coordination, reversal, encapsulation, de-encapsulation and generalization. The definition of construction is presented in Table 2.

Table 2: Definition Construction

Construction Process	***Definition***
Interiorization	Thinking activities to elicit information of given problems
Coordination	Stimulating new processes derived from two or more previous processes
Reversal	The process of recalling knowledge used for continuing the same process.
Encapsulation	Generating a mental object of a mental process
De-encapsulation	Adjusting to a mental object that has been formed in the structure of problems
Generalization	Applying a scheme of wide-ranging problems (in the form of conclusion)

Source: Irfan et al.

The researchers observed the activities of students who completed the test questions provided. In what follows, researchers conducted interviews with selected research subjects. The data in this research were obtained from students' tests after they solved problems in terms of understanding the concept of comparison, and interviews with students conducted by researchers. Each interview in the video and the results of students' written tests were collected. To ensure the validity and reliability of the research, the data analysis was measured using triangulation through displaying data from the video and comparing them with students' written tests data.

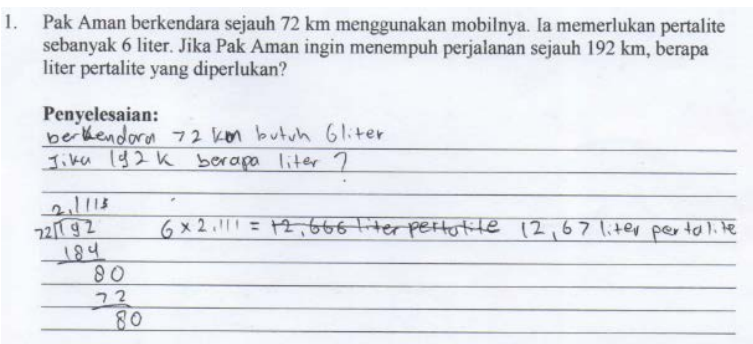
Interview videos were analyzed by investigating details and results of written tests to describe students' mistakes in understanding the concept of proportion. After determining the categories of students' understandings, the next step is to start processing data in relation to the characteristics of students' understandings concerning the concept of proportion. This categorization is an important step in the data analysis because it facilitates a meaningful interpretation of data. This is based on a literature review, identifying each interview answer and the student writing test that represents the related concepts in the literature.

Result

The purpose of this research was to describe the learning journey of students in generating the concept of proportion, and its main focus is the students' thinking process when completing quizzes (as a confirmation for students' understandings after learning things from teachers).

Quiz 1: Direct Proportion

The first quiz is mainly concerned with a value comparison problem regarding the relationship between mileage and fuel requirements. The answer to the participant can be viewed in Figure 1. The participant begins with rewriting information known from the given problem. The question is written like this: "to drive 72 km requires 6 liters of fuel. If the distance is 192 km, how many liters are required?" The answer is based on what students understood after reading the question. In this case, the participant has interiorized:


1. Pak Aman berkendara sejauh 72 km menggunakan mobilnya. Ia memerlukan pertalite sebanyak 6 liter. Jika Pak Aman ingin menempuh perjalanan sejauh 192 km, berapa liter pertalite yang diperlukan?

Penyelesaian:

berkendara 72 km butuh 6 liter
Jika 192 k berapa liter ?
2,111
72 | 192   6 x 2,111 = ~~12,666 liter pertalite~~ 12,67 liter pertalite
184
80
72
80

Figure 1: The Participant Answer to the Quiz 1
Source: Irfan et al.

Some information is formed in a relationship, so that a mental object/formula is set to solve the problem. The answer shows that while driving 72 km, Pak Aman needed 6 liters of pertalite, to drive as far as 192 km would he require more pertalite? In this case, the participant coordinates known information and is able to recall the concept of direct proportion the teacher has taught (reversal). The first step is that 192 km is divided by 72 km. Then the result is multiplied by 6, so that more fuel is required. This is an encapsulation process from the results of previous coordination, but the participant did not explicitly write a formula to calculate the pertalite needed. The participants in the process make a division between 192 and 72 and then the result is multiplied by 6. This is actually a calculation from $\frac{72}{192} = \frac{6}{x}$. Furthermore, the participant generalizes that the need for pertalite to travel 192 km is 12.67 liters. The answer is incorrect. As Figure 1 shows, it can be viewed that the error begins when the participant divides 192 by 72, and the result of the division is 2.111. This is because the participant is wrong in calculating the multiplication between 2 and 72, 144 is written 184, so this affects the subsequent calculation.

Quiz 2: Inverse Proportion

Quiz 2 deals with comparing the value of relationship between the number of workers and the duration of work. The participant solved the problem as shown in Figure 2. Initially, the participant misunderstood the phrase “Pak Tikno added 6 workers.” According to the participant’s understanding, workers who were initially five people became elven people, because there were six additional workers (interiorization). The participant has calculated the results of the understanding. The participant realized the wrong process of interiorization, so he crossed out what had been written. When the participant realized that what he understood was wrong, he then re-read the question.

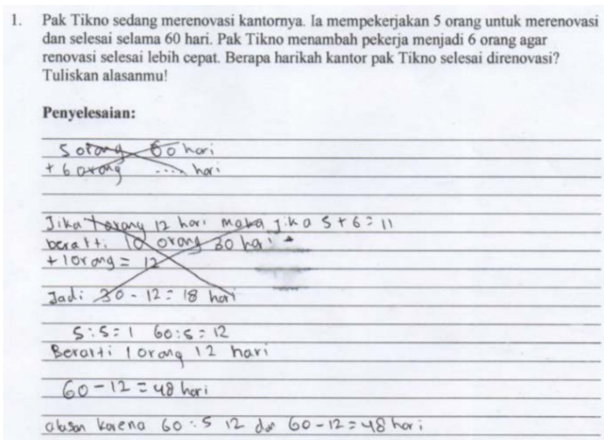
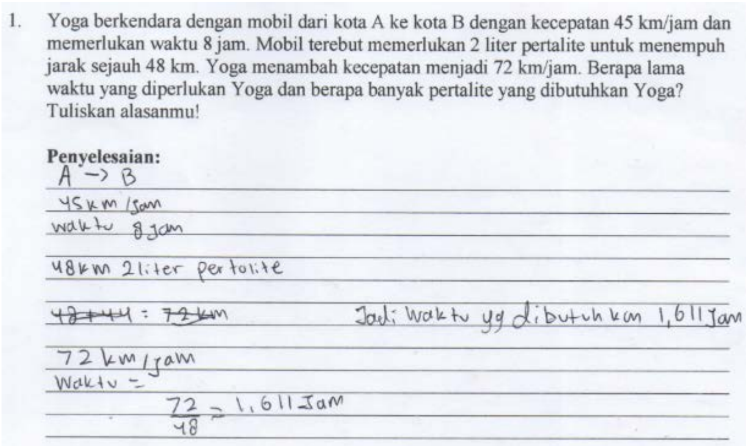


Figure 2: Participant Answer to Quiz 2
Source: Irfan et al.

After re-reading the test item, the participant then writes “and.” Next, the researchers asked, “why do you divide 5 by 5 and why is 60 divided by 5?” Moreover, 5 workers are divided by 5 workers and the result is 1. Then 60 days is divided by 5 workers, and the result is 12. Thus, 1 worker (workers divided by workers) takes 12 days. If we realize the flow of thinking, of course, we will determine the renovation time by adding 60 days to 12 days, with a result of 72 days. However, the participant thinks differently. After getting the 12 days, he decided to use 12 to reduce 60, because in his mind, if there were more workers, then the renovation time was getting faster (the teacher has taught him about the concept). Therefore, he decided to reduce, not to add. In the end, the participant wrote 60 – 12 = 48 days. Then he drew conclusions (generalization) that the renovation time needed 48 days if it was done by 6 workers.

Quiz 3: Direct Proportion and Inverse Proportion

Unlike Quiz 1 and Quiz 2, which only contain one type of comparison, Quiz 3 contains two types of comparisons, namely direct and inverse proportion. Quiz 3 contains the problem of speed and time needed (inverse proportion) and the problem of pertalite needs and the travel distance (direct proportion). The participant's answer is shown in Figure 3.


1. Yoga berkendara dengan mobil dari kota A ke kota B dengan kecepatan 45 km/jam dan memerlukan waktu 8 jam. Mobil terebut memerlukan 2 liter pertalite untuk menempuh jarak sejauh 48 km. Yoga menambah kecepatan menjadi 72 km/jam. Berapa lama waktu yang diperlukan Yoga dan berapa banyak pertalite yang dibutuhkan Yoga? Tuliskan alasanmu!

Penyelesaian:

A -> B
45km/jam
waktu 8jam
48km 2liter pertolite
~~48+44~~ = ~~72km~~
72 km/jam
Waktu =
72/48 = 1,611 Jam
Jadi waktu yg dibutuhkan 1,611 Jam

Figure 3: The Participant's Answer in Quiz 3
Source: Irfan et al.

The participant begins by writing down the information in question. The participant wrote the distance of city A and city B (A→B), the speed of 45 km/h, and 48 km needed 2 liters of pertalite. The interiorization is then coordinated in terms of a mental activity looking for the time needed if the speed changed to 72 km/hr. Participants argued that to find the "time," the "speed" was divided by the "distance." Furthermore, the results of coordination are encapsulated into mental objects in the form of formulas, namely $time = \frac{72}{48}$. Based on this formula, the researchers then asked, "why is 72 divided by 48?" The participant replied, "because I want to find the time needed." The researcher asked again, "is the formula like that?" He answered, "my reasoning is that there is only the speed formula, the distance is divided by time. Therefore, to find time, speed is divided by distance. After calculation, the time taken is 1,611 hours". Furthermore, the participant interpreted results of the calculation at a speed of 72 km/hr, Yoga can arrive at the location for 1,611 hours. The researcher then asked, "why is the time taken only 1,611 hours?" The participant answered, "because the speed increases, so the time is getting shorter."

After solving the problem of the speed and travel time, the participant then resolved the problem of the pertalite needed, as evidenced in Figure 4. To calculate how much pertalite is needed, it is necessary to find the distance first, but the participant did not do that. The participant used a speed of 72 km/h and 45 km/hr. Participants assumed that the driving speed could affect the need for pertalite. The participants wrote 72-45 = 27, 2 liters = 45 km/hr. The participants counted 72-45 interpreted as the difference in speed, which was then used to find pertalite needs. If 2 liters of pertalite are used for a speed of 45 km/h, and a speed of 72 km/h requires 2 liters + 1 liter, and it will be equal to 3 liters of pertalite.

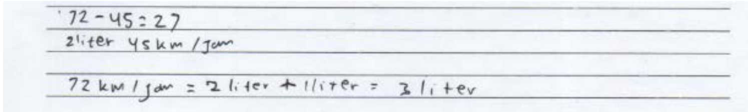


Figure 4: Participants' Answers to the 3-part Pertalite Quiz
Source: Irfan et al.

Discussion

Researchers have examined the problems of proportion. Some researchers discuss students' mistakes when solving problems with them (Fernández et al. 2012; Van Dooren et al. 2009; van Dooren et al. 2010). Their findings include using the concept of addition to solve the problems of proportion, using the concept of proportion to solve non-proportion problems, and distinguishing between direct proportion and inverse proportion. Meanwhile, other researchers are more interested in examining the concept of proportion between them (Boyer and Levine 2012; De Bock, Van Dooren, and Verschaffel 2013; Hilton et al. 2012). In addition, researchers examined the construction of students' understandings of the quiz questions. In terms of students' assignments when solving problems in Quiz 1, the stage of interiorization is marked by the participant written as the information in the quiz question. The results of the interiorization are processed by determining the relationship and the provision of information. The participant encapsulates the results of coordination by forming a mental object in the form of a formula used to solve the problems. In the end, the participant found a solution and interpreted based on the context of the problems (see Figure 5). Errors occur when completing Quiz 1 in the calculation (the division). This results in the wrong final result.

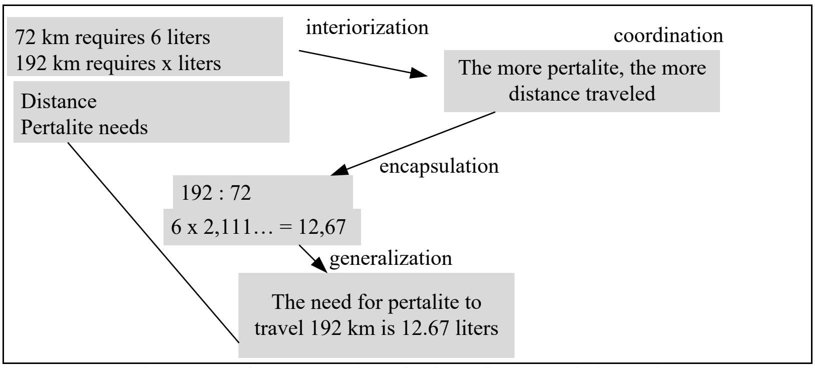


Figure 5: Students' Mental Mechanism when Completing Quiz 1
Source: Irfan et al.

In line with the inverse proportion (Quiz 2), the participant made a mistake during the interiorization. The participant interpreted the sentence incorrectly, "Pak Tikno added workers to 6." The participant interpreted the sentence that Pak Tikno added 6 workers, making a total number of 11 workers. According to (Bloem and La Heij 2003), errors in interpreting sentences are called a semantic interference. From the initial understanding, the participant found that with 11 workers, it took 18 days to renovate the house. Afterward, he read the questions again and found a mistake, then he reinterpreted. And then, the participant connected data from the information, known as the concept of a reverse value. At this stage, there is either coordination or reversal—more and more workers, and the time for home renovation is getting faster. The participant encapsulates the previous process by writing 60: 5 = 12. The point is that if 60 days need 5 workers, it means that 1 worker needs 12 days. This understanding is certainly in contrast to the concept of an inverse proportion. The participant reduces 60 by 12 with the aim of getting the working time with 6 workers. This means that the participant is not able to adjust to the structure of the problem with the concept that has been owned (de-encapsulation). The participant actually knows that if workers increase, the processing time will be faster. This is the reason why he reduced 60 by 12, and he got 48 days, not 60 + 12 = 72 days. In the end, the participant generalized that it took 48 days to renovate a house with 6 workers (see Figure 6).

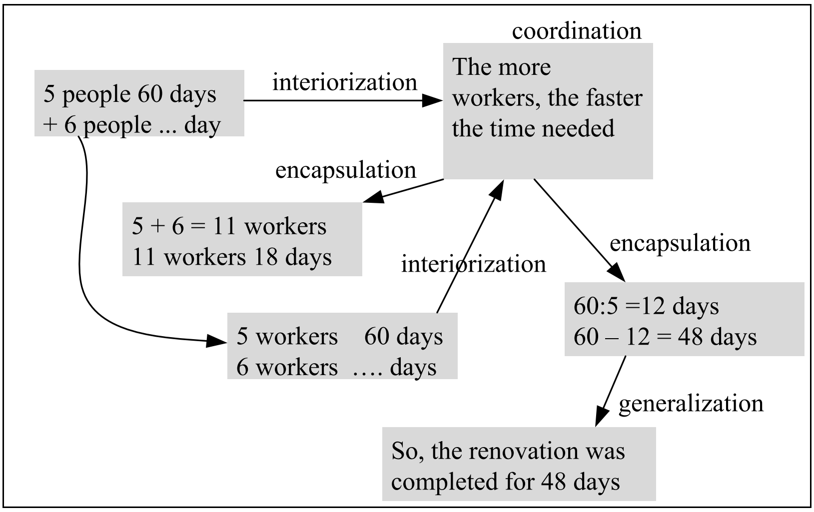


Figure 6: The Students' Mental Mechanism when Completing Quiz 2
Source: Irfan et al.

Quiz 3 contains problems with respect to the direct proportion and inverse proportion. The participant of interiorization is to explore the information in the question and coordinate it, so that the information can be used to solve the problem. In this case, there are two parts of coordination that happen, namely the relationship of speed with fuel requirements and the relationship of speed with the travel time. Furthermore, the participant encapsulates the distance relationship with the needs of pertalite by writing 72-45 = 27; 72 →2 +1 = 3 liters. While encapsulating the relationship of speed with the travel time, the participant wrote $time = \frac{72}{48}$. In the end, the participant concluded that it needed 3 liters of pertalite to travel 72 km / hr and the time needed was 1,611 hours.

In this section, the participant has a problem when coordinating and deciding to calculate the fuel needed by paying attention to the speed, not the distance. The same is true when the participant resolves the time needed. The participant did not pay attention to the first and second speed and the time required. In fact, participants prefer the distance that can be reached with 2 liters of pertalite.

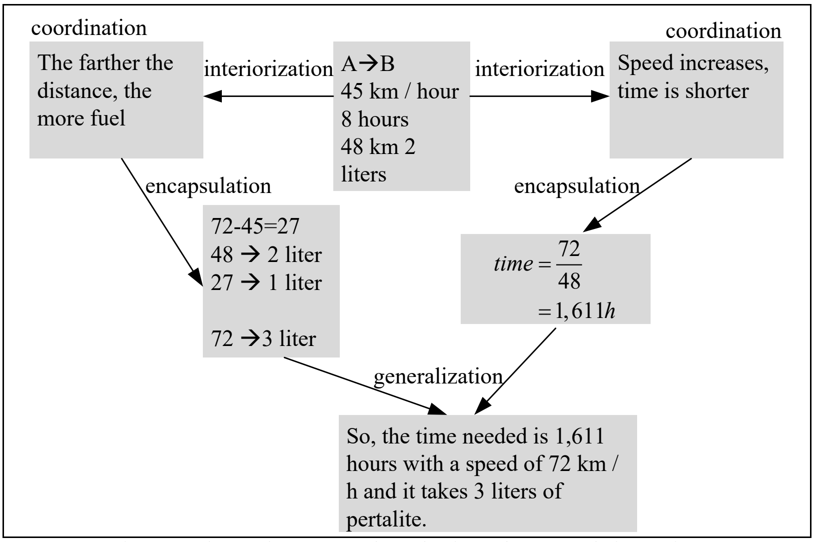


Figure 7: The Students' Mental Mechanism when Completing Quiz 3
Source: Irfan et al.

Indeed, to understand the proportional concept is difficult. The findings of this research are a basic reason why understanding the concept of proportion is not easy. Other researchers show the same understanding including the difficulty of distinguishing between direct proportion and inverse proportion (De Bock, Van Dooren, and Verschaffel 2013; Irfan et al. 2019, 2018), the difficulty of distinguishing between proportion and non-proportion problems (Irfan et al. 2019; Fernández et al. 2012; Van Dooren et al. 2009), and the difficulty in making a proportion question (Modestou et al. 2008).

To be able to understand the concept of proportion, students must be able to understand each part of learning. If the student does not understand each part of it, then in the next learning session, students will not understand, and one of the consequences is that students cannot solve the problem correctly. The concept of proportion is formed from several previous materials, including division, fraction, multiplication, and ratio. To be able to master the concept of proportion, these materials must be understood first.

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